NATURAL OSCILLATIONS OF A GAS FLOWING ABOUT A LATTICE OF PLATES

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ABSTRACT. Oscillations of a gas flow around a lattice of plates were studied with the application of the contraction method for solution. Also considered were natural oscillations of gas in an infinite plane, channelling them annularly, given the condition of spatial periodicity of flow.

The contraction method is used to solve the problem of the natural oscil- /68* lations of a gas flowing about a lattice of plates. In addition, under the condition of spatial periodicity of the flow, the natural oscillations of a gas in an infinite plate are considered, modeling the natural oscillations of a gas in an annular channel.

Interest in studying this problem was evoked by the results of a series of experiments on the oscillations of a lattice of plates in a subsonic gas flow [1-4]. It was demonstrated in these papers that for a certain combination of lattice parameters and parameters of incident flow, the non-stationary aerodynamic characteristics of the plates are highly dependent on these parameters. From the physical standpoint, such phenomena found their explanation in the acoustic resonance of the excited gas, produced by oscillations of the profile, with corresponding oscillations of the gas in the lattice region in question. It was noted in this regard that resonance regimes show a significant reduction of aerodynamic deformation of the lattice oscillations.

This fact was mentioned in [5] in a study of the osci-lations in axial compressors. It was in this paper that the relationships determining the values of the natural oscillations of a gas in an annular channel in a circular direction were first stated. The same relationships, but in a different form and obtained by other methods, are also found in [6, 7].

1. Let us begin by considering the problem of the natural oscillations of a gas flow in an infinite plane. The periodic solutions of this problem will be a model of the natural oscillations of a gas in an annular channel. The

*Numbers in the margin indicate pagination in the foreign text.

mathematical problem amounts to a determination of the solution of the equation, limited to the entire plane, for the amplitude of the non-stationary component of the potential of the flow rate ϕ. In a dimensionless Cartesian system of coordinates x and y, relative to some characteristic length c, it has the form

$$(1 - M^2)\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} - 2kMj\frac{\partial\varphi}{\partial x} + k^2\varphi = 0$$

$$\varphi' = \varphi(x, y)\,e^{j\omega t}, \qquad M = \frac{U}{a}, \qquad k = \frac{\omega c}{a} \tag{1.1}$$

Here ω is the frequency of oscillation of the gas, U is the speed of the undisturbed flow of gas along the x axis, and a is the speed of sound in an undisturbed flow.

Rotating axes x and y through angle β, we shift to new dimensionless coordinates ξ, η:

$$\xi = x\cos\beta - y\sin\beta, \qquad \eta = x\sin\beta + y\cos\beta \tag{1.2}$$

In this system of coordinates, (1.1) is changed to read as follows:

$$\Delta\varphi - M^2\cos^2\beta\,\frac{\partial^2\varphi}{\partial\xi^2} - M^2\sin^2\beta\,\frac{\partial^2\varphi}{\partial\eta^2} - M^2\sin 2\beta\,\frac{\partial^2\varphi}{\partial\xi\,\partial\eta} -$$
$$- 2kMj\cos\beta\,\frac{\partial\varphi}{\partial\xi} - 2kMj\sin\beta\,\frac{\partial\varphi}{\partial\eta} + k^2\varphi = 0 \tag{1.3}$$

We shall seek the general solution of (1.3) in the class of functions which /69 are periodic in the direction η, with a period L which is equal to the length of the circumference of the corresponding annular channel, also relative to c. Then the function ϕ is represented by a Fourier series:

$$\varphi = \sum_{n=-\infty}^{\infty} f_n(\xi)\exp\frac{j2\pi n\eta}{L} \tag{1.4}$$

and the partial periodic solution of (1.3) can be expressed as follows:

$$\varphi_n = \exp(\lambda_n\xi + j\,2\pi n\eta\,/\,L) \tag{1.5}$$

Substituting (1.5) in (1.3), we obtain the characteristic equation for determining λ_n, whose solution has the form

$$\lambda_n = j\lambda_{1n} \pm \lambda_{2n}, \qquad \lambda_{1n} = \frac{M\cos\beta}{1 - M^2\cos^2\beta}\left[k + \frac{2\pi n}{L}M\sin\beta\right]$$

$$\lambda_{2n} = \frac{1}{1 - M^2\cos^2\beta}\left[\left(\frac{2\pi n}{L}\right)^2(1 - M^2) - 2k\frac{2\pi n}{L}M\sin\beta - k^2\right]^{1/2} \tag{1.6}$$

Thus, the general solution of (1.3) which satisfies the condition of periodicity in the direction of axis η has the form

$$\varphi = \sum_{n=-\infty}^{\infty} \exp\frac{j2\pi n\eta}{L} \left[a_n \exp\left(j\lambda_{1n} + \lambda_{2n}\right)\xi + b_n \exp\left(j\lambda_{1n} - \lambda_{2n}\right)\xi \right] \tag{1.7}$$

If the subradical expression in (1.6) for λ_{2n} is less than or equal to zero, the corresponding term of the series (1.7)

$$\varphi_n = \exp\left[j\left(\lambda_{1n}\xi + 2\pi n\eta / L\right)\right] \cos\left[\lambda_{2n}'\left(\xi + \delta\right)\right] \quad \left(\lambda_{2n} = j\lambda_{2n}'\right) \tag{1.8}$$

where δ is some arbitrary number, will be the natural function of the problem in question. It satisfies (1.3) and is bounded in the entire plane. Similar solutions of the Helmholtz equation for an infinite plane are given (for example) in the book by Courant and Gilbert [8].

Let us extract from (1.8) the factor $\exp\left(j\lambda_{1n}\xi\right)$ which characterizes the drift of the gas disturbance in the flow in the direction ξ, assuming $\phi_n = \exp\left(j\lambda_{1n}\xi\right)\phi_n^*$. Then the function $\phi_n^* \exp\left(j\omega t\right)$ will be a superposition of two traveling waves, propagating in directions which are symmetrical relative to axis η. In the boundary case, when $\lambda_{2n} = 0$, the function $\phi_n^* \exp\left(j\omega t\right)$ is a traveling wave propagating only in the direction of axis η.

Let us introduce the designations
$$L = Nh, \qquad n = n_1 N + m, \qquad \mu = 2\pi m / N$$
$$(m = 0, 1, \ldots, N-1; \; n_1 = 0, \pm 1, \pm 2, \ldots) \tag{1.9}$$
where N is some natural number. Then
$$\frac{2\pi n}{L} = \frac{2\pi n_1 + \mu}{h} \tag{1.10}$$
and we can see that the condition $\lambda_{2n} = 0$ coincides with the condition of acoustic resonance of the natural oscillations of a gas in an infinite plane with disturbances evoked by a pulsating chain of dipoles [7] arranged along the axis η with intervals h:
$$2\pi n_1 + \mu = \frac{kh}{1 - M^2}\left[M\sin\beta \pm \sqrt{1 - M^2\cos^2\beta}\right] \quad (n_1 = 0, \pm 1, \ldots) \tag{1.11}$$

From the physical standpoint, this means that a chain of dipoles, radiating a disturbance, resonates with those natural oscillations of a gas which do not contain waves approaching from left or right, from infinity to the η axis.

As indicated in [2-5], condition (1.11) also determines some properties of the non-stationary flow of a gas through the plane lattice of a profile.

In this case the parameter h is the dimensionless interval of the lattice, relative to the semichord of the profile c, and the parameter β is the loss angle of the lattice. Like the pulsating chain of dipoles, the lattice is the source of the excited gas; with synchronous oscillations of its profile having identical amplitudes and a constant phase shift μ, it stimulates a corresponding form of natural oscillation in the infinite plane. The interaction of the lattice profiles with the gas decreases sharply, and the aerodynamic damping of the oscillations of the profile also decreases. Defining this phenomenon in flow around a lattice as acoustic resonance, we must mention its definite conditionality, since the natural oscillations of a gas in a "lattice" region in the general case do not coincide with the natural oscillations we have considered.

2. In the case of flow around a lattice of plates by a subsonic stream of gas, the problem of the natural values consists in the determination of the non-trivial solution of (1.3) under the condition of limitation of the lattice to an infinite extent behind and in front of the lattice (Figure 1).

$$\varphi < \infty \text{ when } |\xi| \to \infty \tag{2.1}$$

and with a uniform condition of non-flow of the gas through the plates:

$$\partial \varphi / \partial y = 0 \text{ when } y = nh \cos \beta$$
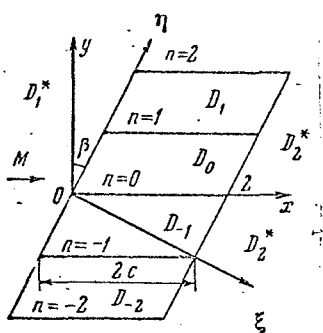$$nh \sin \beta < x < nh \sin \beta + 2 \tag{2.2}$$

Figure 1.

We shall limit ourselves to a consideration of those flows in which there are no vortical traces beyond the plates. We shall seek these solutions with the aid of the method of contraction [9]. According to this method, the flow region is broken up into regions D_1^* and D_2^* (located to the left and right, respectively, of the lattice and bounded by lines joining the leading and trailing edges of the plates) and region D_n (enclosed between the plates (Figure 1)). In regions D_1^* and D_2^*, in accordance with the concept of

periodic functions (1.7), the most general expression for the natural functions, with consideration of substitution (1.11) has the form

$$\varphi = \sum_{m=0}^{N-1} \exp\left(j\mu\,\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} \exp\left(j2\pi n\,\frac{\eta}{L} + \lambda_{1n}\xi\right) [a_{mn_1}e^{\lambda_{2n}\xi} + b_{mn_1}e^{-\lambda_{2n}\xi}] \tag{2.3}$$

It will be shown later on that any desired function in these regions is described by one of the terms of the sum over m. In addition, from condition (2.1) and from the condition of the absence of waves arriving from infinity, it follows that the coefficients a_{mn1} are equal to zero in region D_2^* and coefficients b_{mn1} are equal to zero in region D_1^*. Therefore we assume that for region D_1^*

$$\varphi_1^* = \exp\left(j\mu\,\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} a_n \exp\left[(j\lambda_{1n} + \lambda_{2n})\,\xi + j2\pi n\,\frac{\eta}{h}\right] \tag{2.4}$$

and for region D_2^*

$$\varphi_2^* = \exp\left(j\mu\,\frac{\eta}{h}\right) \sum_{n=-\infty}^{\infty} b_n \exp\left[(j\lambda_{1n} - \lambda_{2n})\,(\xi - 2\cos\beta) + j2\pi n\,\frac{\eta}{h}\right] \tag{2.5}$$

In (2.4) and (2.5) and the equations that follow, the subscript 1 in n_1 in constants a_{mn1} and b_{mn1} is omitted.

The functions ϕ_1^* and ϕ_2^* and their first derivatives will be continuous in regions D_1^* and D_2^* with the possible exception of the values $\eta = sh$ and $\eta = 2\sin\beta + sh$ ($s = 0, \pm 1, \pm 2, \ldots$) at their boundaries, corresponding to the coordinates of the edges of the profiles. From thin-wing theory we know that at these points the derivative of the function of the speed potential may have a singularity of the type $(r - r_s)^{-1/2}$, where r_s is the coordinate of one of the edges of the s-th profile and r is the radius vector of the flowing coordinate. However, regardless of these singularities, the coefficients $a_n{}'$ and $b_n{}'$ of the series for derivatives of the functions ϕ_1^* and ϕ_2^*, analogous to the series of expressions (2.4) and (2.5), tend toward zero at $n \to \infty$ so that

$$a_n{}'\big|_{n\to\infty} = \frac{2}{h}\int_0^h \left[\frac{c_0}{\sqrt{\eta\,(h-\eta)}} + f(\eta)\right]\exp\left(j2\pi n\,\frac{\eta}{h}\right)d\eta = \frac{2\pi c_0}{h}J_0\,(nh) + o(n^{-1}) \to 0$$

Here in the form $c_0/\sqrt{\eta(h-\eta)}$ we have isolated the term with the singularity from the derivative functions $\phi_1^*(0, \eta)$, $\phi_2^*(2\cos\beta, \eta)$. Consequently, the series for the first derivative functions of the speed potential converge at any value of η, excluding the coordinates of the edges of the profiles.

In regions D_n the general expression of the natural functions will be determined by the solution of mixed problems of the type

$$\varphi_n = \varphi_1^* \quad \text{with } \xi = 0, \quad \varphi_n = \varphi_2^* \quad \text{with } \xi = 2\cos\beta$$
$$\partial\varphi_n / \partial y = 0 \text{ with } y = nh\cos\beta, \quad y = (n+1)h\cos\beta \qquad (2.6)$$

The functions ϕ_n will be sought in the form of an infinite series of solutions of (1.1), each of which satisfies the condition of not flowing through. Considering that the first two conditions (2.6) differ for the various regions only by the factor exp $(jn\mu)$, the general expression for the function ϕ_n may be represented as follows:

$$\varphi_n = e^{j(\sigma x + n\mu)} \sum_{m=0}^{\infty} [c_m e^{\lambda_m(x-2)} + d_m e^{-\lambda_m x}] \cos\left[\pi m \left(\frac{y}{h\cos\beta} - n\right)\right]$$

$$\lambda_m = \frac{1}{1-M^2}\left[\left(\frac{\pi m}{h\cos\beta}\right)^2 (1-M^2) - k^2\right]^{1/2}, \qquad \sigma = \frac{kM}{1-M^2} \qquad (2.7)$$

The constant a_n and b_n of functions (2.4) and (2.5) as well as the constants c_m and d_m of function (2.7) will be determined in accordance with the method of contraction from the condition of continuity of the desired function and its normal derivative on lines $\xi = 0$ and $\xi = 2\cos\beta$, excluding, perhaps, the points which ocrrespond to the coordinates of the edges of the profile. This automatically satisfies the first two conditions (2.6) for function (2.7).

It should be mentioned that it is sufficient to carry out the contraction of functions (2.4) and (2.5) with function (2.7) in the space of only one h interval. On the remaining portions of the lines $\xi = 0$ and $\xi = 2\cos\beta$ it is lacking due to the condition of periodicity. But since expressions (2.4) and (2.5) describe the arbitrary function of the desired solution in the space of one interval, it is possible to state that summation over the

subscript *m* in (2.3) does not give a more general idea of the desired solution and may be eliminated.

Thus, equating functions (2.4) and (2.5) and their derivatives in the direction ξ to the function (2.7) and its derivative on lines $\xi = 0$ and $\xi = 2 \cos \beta$, respectively, we will obtain four relationships which link the unknown constants.

For the sake of brevity, we shall write only two of these equations which satisfy the conditions of continuity on the line $\xi = 0$.

$$
\exp \frac{j\mu\eta}{h} \sum_{n=-\infty}^{\infty} a_n \exp \frac{j2\pi n\eta}{h} =
$$

$$
= \sum_{m=0}^{\infty} [c_m e^{\lambda_m(\eta \sin\beta-2)} + d_m e^{-\lambda_m \eta \sin\beta}] e^{j\sigma\eta\sin\beta} \cos \frac{\pi m\eta}{h}
$$

$$
\exp \frac{j\mu\eta}{h} \sum_{n=-\infty}^{\infty} a_n (j\lambda_{1n} + \lambda_{2n}) \exp \frac{j2\pi n\eta}{h} = \tag{2.8}
$$

$$
= \sum_{m=0}^{\infty} e^{j\sigma\eta\sin\beta} \left\{ \cos\beta \left[c_m (\lambda_m + j\sigma) e^{\lambda_m(\eta\sin\beta-2)} + d_m (-\lambda_m + j\sigma) e^{-\lambda_m\eta\sin\beta} \right] \times \right.
$$

$$
\left. \times \cos\frac{\pi m\eta}{h} + \frac{\pi m}{h}\tan\beta \left[c_m e^{\lambda_m(\eta\sin\beta-2)} + d_m e^{-\lambda_m\eta\sin\beta} \right] \sin\frac{\pi m\eta}{h} \right\}
$$

To determine the unknown constants from system (2.8) we will shift to an infinite system of algebraic equations. With this goal in mind, the relationships obtained are multiplied on the left and right by the function $\exp [-j (2\pi n + \mu) \times \eta/h] (n = 0, 1, 2,...)$ and integrated over η from 0 to h. We will then have

$$
a_n = \sum_{m=0}^{\infty} [c_m e^{-2\lambda_m} A_{nm} + d_m B_{nm}].
$$

$$
(j\lambda_{1n} + \lambda_{2n}) a_n = \sum_{m=0}^{\infty} \left\{ c_m e^{-2\lambda_m} \left[(\lambda_m + j\sigma)\cos\beta - \frac{(\pi m)^2\tan\beta}{h\theta_{nm}'} \right] A_{nm} + \right. \tag{2.9}
$$

$$
\left. + d_m \left[(-\lambda_m + j\sigma)\cos\beta - \frac{(\pi m)^2\tan\beta}{h\theta_{nm}''} \right] B_{nm} \right\}
$$

Here

$$A_{nm} = \frac{\theta_{nm}'}{\theta_{nm}'^2 + (\pi m)^2} [(-1)^m \exp(\theta_{nm}') - 1]$$

$$B_{nm} = \frac{\theta_{nm}''}{\theta_{nm}''^2 + (\pi m)^2} [(-1)^m \exp(\theta_{nm}'') - 1]$$

$$\theta_{nm}' = j(\sigma h \sin\beta - 2\pi n - \mu) + \lambda_m h \sin\beta$$

$$\theta_{nm}'' = j(\sigma h \sin\beta - 2\pi n - \mu) - \lambda_m h \sin\beta$$

From each pair of the system (2.9) it is easy to exclude the constants a_n. Performing similar operations for the relationships that fulfill the conditions of continuity on the line $\xi = 2 \cos \beta$, the system of equations for determining the unknown constants c_m and d_m is obtained in the following form:

$$\sum_{m=0}^{\infty} \left\{ c_m \left[\cos\beta(\lambda_m + j\sigma) - \frac{(\pi m)^2 \tan\beta}{h\theta_{nm}'} - (j\lambda_{1n} + \lambda_{2n}) \right] A_{nm} e^{-2\lambda_m} + \right.$$

$$\left. + d_m \left[\cos\beta(-\lambda_m + j\sigma) - \frac{(\pi m)^2 \tan\beta}{h\theta_{nm}''} - (j\lambda_{1n} + \lambda_{2n}) \right] B_{nm} \right\} = 0$$

$$\sum_{m=0}^{\infty} \left\{ c_m \left[\cos\beta(\lambda_m + j\sigma) - \frac{(\pi m)^2 \tan\beta}{h\theta_{nm}'} - (j\lambda_{1n} - \lambda_{2n}) \right] A_{nm} + \right.$$

(2.10)

$$\left. + d_m \left[\cos\beta(-\lambda_m + j\sigma) - \frac{(\pi m)^2 \tan\beta}{h\theta_{nm}''} - (j\lambda_{1n} - \lambda_{2n}) \right] B_{nm} e^{-2\lambda_m} \right\} = 0$$

$$(n = 0, \pm 1, \pm 2, \dots)$$

Since system (2.10) will be uniform, the existence of its non-trivial solution is possible only under the condition of equality to zero of the determinant, composed of the coefficients at unknown constants. Thus, the problem under consideration becomes one of determining the natural values of an infinite system of algebraic equations (2.10).

It should be mentioned that the approximate values of the natural numbers of the system (2.10), with the accuracy given in advance, can be determined from the truncated system, i.e., we shall use the method of reduction for its solution. In the general case, however, even this problem poses significant computational difficulties. For the sake of illustration, we have listed below an analysis of the natural oscillations of a gas in a lattice

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area, using the simplest example of flow about a lattice without removal.

3. In the case of a lattice without removal ($\beta = 0$) the infinite system of algebraic equations is simplified considerably. In fact, at $\beta = 0$ the coefficients $A_{n_m} = B_{n_m}$, $\theta_{n_m}' = \theta_{n_m}''$, $\sigma = \lambda_{1n}$ and the system of equations (2.10) assumes the form

$$\sum_{m=0}^{\infty} [c_m (\lambda_m - \lambda_{2n}) e^{-2\lambda_m} - d_m (\lambda_m + \lambda_{2n})] A_{nm} = 0$$

$$\sum_{m=0}^{m} [c_m (\lambda_m + \lambda_{2n}) - d_m (\lambda_m - \lambda_{2n}) e^{-2\lambda_m}] A_{nm} = 0$$

$$(n = 0, \pm 1, \pm 2 \ldots)$$

From this it follows that $c_m = d_m$, and system (2.10) thus assumes the form

$$\sum_{m=0}^{\infty} c_m [\lambda_m + \lambda_{2n} - e^{-2\lambda_m} (\lambda_m - \lambda_{2n})] A_{nm} = 0 \quad (n = 0, 1, 2 \ldots) \qquad (3.1)$$

Note first of all that at $\mu = 0$ the coefficients $A_{n_m} = 0$ if $m \neq 2n$. But since $\lambda_m = \lambda_{2n}$ at $m = 2n$, it follows from system (3.1) that all constants c_m and d_m (and consequently a_n and b_n as well) in this case are equal to zero. Thus, at $\mu = 0$ there is no non-trivial solution of system (3.1). Furthermore, it is easy to see that if the value λ_m becomes zero at some fixed value of m, the two columns of the determinant of the system will converge.

Thus, the condition $\lambda_m = 0$ determines the composition of the parameters

$$k = \frac{\pi m}{h} \sqrt{1 - M^2} \quad (m = 1, 2, \ldots) \qquad (3.2)$$

at which a gas flowing about a lattice of plates without removal can complete natural oscillations. The natural functions of (1.1) corresponding to these oscillations have the form

$$\varphi = \exp (j\sigma x) \cos (m\pi y / h) \qquad (3.3)$$

Note that (3.2) coincides with relationship (1.11) at $\beta = 0$ and $\mu = \pi$, i.e., in this partial case the natural oscillations of the gas in the lattice region coincide with the natural oscillations of the gas in a dimensionless plane.

In the case of induced oscillations of the lattice at a frequency that satisfies condition (3.2), the period of the oscillations is a multiple of the time required for the wave of disturbance to travel from some point on the profile to the corresponding point of the adjacent profile and, being reflected, to return to the original point. If the vibrations of the adjacent profiles are then completed in antiphase, the disturbances evoked by each of the profiles will be superposed and acoustic resonance will occur.

However, the natural oscillations of a gas that arise under condition (3.2) do not exhaust the entire spectrum of natural oscillations which are of practical interest for the case of a lattice of plates without removal. The natural oscillations which are considered include only oscillations in the transverse direction (in the direction of the front of the lattice). In accordance with the results known from acoustics for open resonators [10], we can expect natural oscillations of the gas in the longitudinal direction as well, since the channels between the profiles are essentially resonators of this kind.

In the first approximation the natural frequencies of such oscillations will be determined from condition [6]

$$ k = (1 - M^2)\, \pi m\, /\, 2 \qquad (m = 1, 2, \ldots) \tag{3.4} $$

corresponding to the case of complete reflection of a plane wave with slight disturbance from the open ends.

In reality, the plane wave is not reflected completely from the open end, but interacts with the surrounding space, including the adjacent channels between profiles. Therefore, the corresponding natural function will not be a simple plane wave, localizing itself in one channel, but some complex function in the entire flow plane, allowing for this interaction. The natural frequencies of the oscillations will differ from the values determined by (3.4).

Let us introduce the parameter α_m, taking into account the correction at the open end in (3.4), since the value of the introduced natural frequency of the longitudinal oscillations of the gas is

$$k_m = (1 - M^2)\, \pi m\, (1 + \alpha_m)/2 \qquad (m = 1, 2, \ldots) \qquad (3.5)$$

The value of the parameter α_m, as well as the natural function of the desired oscillations, can be determined with the aid of the solution of system (3.1).

Let us consider the example of the calculation for $m = 1$, $\mu = \pi$. In this case, the expressions for the natural functions (2.4), (2.5) and (2.7) are converted to the form

$$\varphi_1^* = \sum_{n=0}^{\infty} a_n \exp\left[(j\sigma + \lambda_{2n})x\right] \sin \frac{(2n+1)\pi y}{h}$$

$$\varphi_2^* = \sum_{n=0}^{\infty} b_n \exp\left[(j\sigma - \lambda_{2n})(x-2)\right] \sin \frac{(2n+1)\pi y}{h}$$

$$\varphi_n = \sum_{m=1}^{\infty} e^{j\sigma x}\left[c_m \exp \lambda_m (x-2) + d_m \exp(-\lambda_m x)\right] \cos \frac{2\pi m y}{h}$$

$$\lambda_m = \frac{1}{1-M^2}\left[\left(\frac{2\pi m}{h}\right)^2 (1 - M^2) - k^2\right]^{1/2},$$

$$\lambda_{2n} = \frac{1}{1-M^2}\left[\left(\frac{2n+1}{h}\right)^2 \pi^2 (1 - M^2) - k^2\right]^{1/2}$$

Following the method of reduction, system (3.1) was truncated in calculation to $N = 30$ equations. The equality to zero of the determinant of the truncated system at fixed values of M and h was viewed as an approximate condition for the determination of the parameter α_1.

The results of the calculation of parameter α_1 as a function of the dimensionless interval h at values of the number $M = 0$, 0.5 and 0.7 are shown in Figure 2. Analyzing this curve, it is interesting to note that $\alpha_1 \to 0$ at $h \to 0$. From the physical point of view, this result can be explained by the fact that with an increase in the length of the channel, the amount of kinetic energy of the disturbed gas radiated from the open end decreases relative to the energy of the gas oscillating within the channel; at the limit (for an

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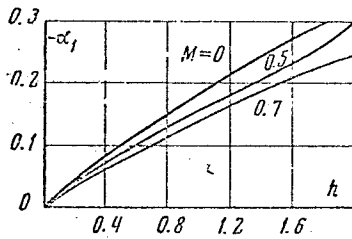infinitely long channel) it tends toward zero.

Figure 2.

Let us list the results of the calculation of the values of the first ten desired coefficients a_n and c_n normalized over a_0, at $h = 1$ and $M = 0.7$.

$n =$		0	1	2	3	4
$a_n =$		1.0	0.1660	0.0761	0.0462	0.0320
— $c_n =$	—0.69— j	3.12	0.2480	0.0898	0.0489	0.0316
$n =$	5	6	7	8	9	10
$a_n =$	0.0241	0.0191	0.0157	0.0132	0.0114	0.0101
$c_n =$	0.0224	0.0168	0.0132	0.0107	0.0089	0.0075

These values, accurate to the third decimal place, coincide with the
coefficients calculated from a system truncated at 20 equations, which is an indication of the good agreement of the method of reduction in this case. However, as far as we can judge from the decrease of the coefficients, the agreement of the derivatives of the desired function is poor.

This fact is illustrated by Figure 3, which shows the dependencies of the normal derivatives of functions ϕ_0 (solid curve) and ϕ_1^* (dashed curve) upon y on the line $x = 0$. It is clear from these curves that the normal derivatives of the desired functions to the left and right of the line of contraction differ from one another with respect to the magnitude of the order of error of their approximation by a finite trigonometric series. (Contraction of actual functions ϕ_0 and ϕ_1^* in the case in question occurs with an accuracy up to 3-4 significant figures). In accordance with the order of descent of the coefficients a_n and b_n at the edges of the plates, there is a clearly pronounced exclusion of the derivatives from the desired functions. Depending on the structure of the solution, these features are found at both ends of the plate.
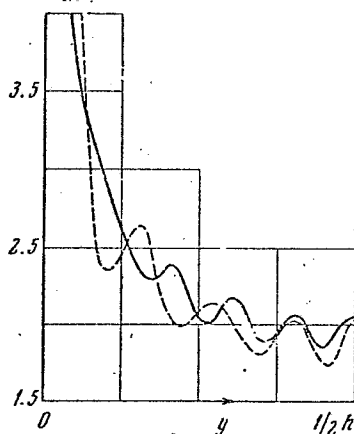
Figure 3.

It should be noted that such solutions evidently exist physically only when $M = 0$.. In a flow with $M \neq 0$, the maximum practical interest is produced by solutions in a class with limited derivatives at the trailing edges of the plates. In the general case, these solutions must be sought with consideration of the vortical traces and the corresponding natural values among the complex numbers.

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